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Acoustically controlled heat transfer of ferromagnetic fluid

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Abstract

The acoustically controlled heat transfer enhancement of a ferromagnetic fluid in an external magnetic field is studied in this paper. The analytical expression for the effective thermal diffusivity is obtained. The numerical analysis and experimental results are discussed. Published by Elsevier Science Ltd.

Keywords: Heat transfer enhancement; Acoustic field control; Ferromagnetic fluid; Magnetic field

1. Introduction

The heat transfer characteristics of a ferromagnetic fluid are of interest in many branches of science and engineering [1,2]. The enhanced-heat conduction in a ferromagnetic fluid may be effectively controlled by a magnetic field [3–5]. So far, attempts to study the heat transfer of ferromagnetic fluid have been limited only to the method by magnet control. It is well known from the investigations of Kurzweg and his coworkers [6–8], Nishio et al. [9] and Gau et al. [10] that heat transfer of a fluid can be enhanced by the fluid oscillating or reciprocating inside a tube.

It is the purpose of this paper to study the acoustically controlled heat transfer of a ferromagnetic fluid in a steady magnetic field. The technique is of the great application potential, e.g. the thermoacoustic heat-transport tube may be made by using the technique, the acoustically controlled ferromagnetic fluid can act as the working fluid of a liquid-sodium thermoacoustic engine or a heat-exchanger in a magnetic refrigerator, and so on.

In this paper, the "Oscillation-enhanced heat transfer" of a ferromagnetic fluid in an external magnetic field is examined. The analytical expression for the effective thermal diffusivity is investigated. The numerical analysis and the experimental results are discussed.

In this paper, we assume that: (1) the radial gradient of the pressure is neglected throughout the tube; (2) in a quiescent acoustic field, the time average of the oscillating velocity equals to zero and the time average of the pressure is a constant; (3) the magnetization is aligned with the magnetic field.

2. Experimental apparatus

Fig. 1 depicts a schematic diagram of the experimental apparatus used in our study. The test section is a horizontal tube made of acrylic resin. The horizontal tube inner radius is l0 mm and length is 150 mm. The copper tubes whose thickness is 0.5 mm with the same inner radius are mounted at the ends of the acrylic resin tube. The copper tube at the right side is heated uniformly by an electric resistance wire. The wire is treated with insulation paint. The heat flow can be calculated from the corresponding electric power. A Helmholtz coil is employed to generate an axial external magnetic field. The temperatures at the hot and cold ends were measured by copper-constantan thermocouples having wire diameter of 0.1 mm. A pressure wave generator generates an acoustic field, which produces one-dimensional laminar sinusoidal oscillations of the fluid. The sample is a petroleum-based ferromagnetic fluid in the tube.

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Nomenclature

c	heat capacity
f_v	viscous dissipation function
f_α	thermal dissipation function
$g(r)$	temperature distribution function
g_r, g_i	real and imaginary part of the $g(r)$
$\vec{H}_e$	external magnetic field
$\vec{H}_1$	demagnetizing field
i	$\sqrt{-1}$
J_0	Bessel function of first kind of order zero
k	thermal conductivity
k_1	effective pyromagnetic coefficient
k_e	effective thermal diffusivity
l	tube length
M	magnetization
p	pressure
Pr	Prandtl number, v/α
r	radial coordinate
r_0	tube radius
t	time
t_v	$\sqrt{\omega\tau_v}$
t_α	$\sqrt{\omega\tau_\alpha}$
T	temperature
T_0	mean temperature
T_h, T_c	temperatures at hot and cold ends
ΔT	temperature difference
u	velocity
u_0	maximum axial velocity
$u(r)$	velocity distribution function
u_r, u_i	real and imaginary part of the $u(r)$
z	axial coordinate
Δz	tidal displacement
α	thermal diffusivity, $k/\rho_0 c$
λ	non-dimensional thermal diffusivity
τ_v	viscosity relaxation time, $r_0^2/2v$
τ_α	thermal relaxation time, $r_0^2/2\alpha$
μ_0	permeability of free space
γ	axial temperature gradient
η	viscosity
ρ_0	mean density
v	kinematic viscosity, η/ρ_0
θ	phase shift between pressure and velocity
χ	susceptibility
ω	angular frequency
ω'	complex angular frequency

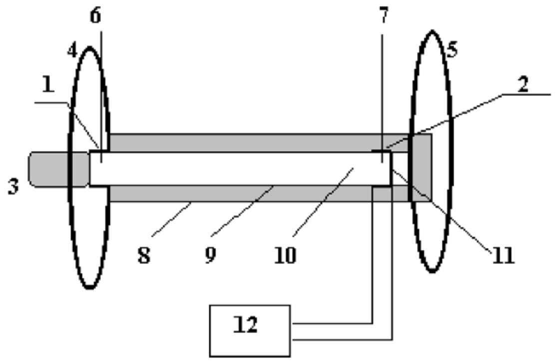

Fig. 1. A schematic diagram of the experimental apparatus: 1. cold copper tube; 2. hot copper tube heated by the electric resistance wire treated with insulation paint; 3. pressure wave generator; 4, 5. Helmholtz coil; 6, 7. copper–constantan thermocouples; 8. "Durkee" heat insulation layer; 9. acrylic resin tube; 10. ferromagnetic fluid; 11. flexible membrane; 12. D.C. generator connected by the electric resistance wire.

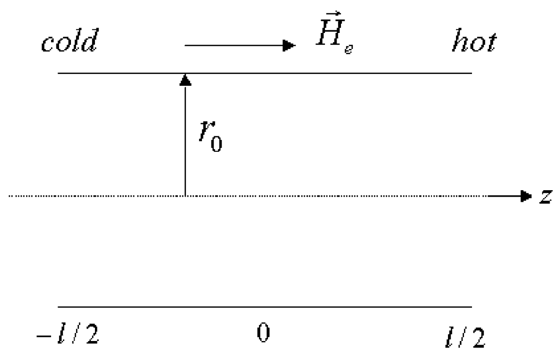

Fig. 2. Geometry used for analysis.

3. Theoretical analysis

As shown in Fig. 2, we study the heat transport phenomena of incompressible oscillating flow of a ferromagnetic fluid in the presence of an axial external magnetic field $\vec{H}_e$. A constant axial temperature gradient $\gamma = (T_h - T_c)/l$ is superimposed on the fluid, where T_h and T_c are the temperatures of the hot and cold ends at $z = l/2$ and $z = -l/2$, respectively, with l being the channel length.

A velocity oscillating induces a temperature oscillating and the temperature oscillating induces a magnetic oscillating, so the magnetic field within the ferromagnetic fluid is related to the velocity oscillating. In the presence of an external field $\vec{H}_e$, the internal field is [11]

$$\vec{H} = \vec{H}_e + \vec{H}_1, \tag{1}$$

where $\vec{H}_1$ is the demagnetizing field. The magnetic field is linearized about velocity oscillating u by the expression [12,13]

$$\vec{H}_1 = \vec{H}_{10} + \vec{H}' \tag{2}$$

with

$$\vec{H}' = \left(\vec{H}_0 + \frac{k_1 \gamma z}{1+\chi}\right)\vec{u},$$

where H_{10} is regarded as a constant and $\vec{H}_{10} + H_0\vec{u}$ is the corresponding value of the $\vec{H}_1$ at $z = 0$, χ and k_1 are the susceptibility and the effective pyromagnetic coefficient, respectively, H', which is assumed to be small in comparison to H_{10}, is the magnetic disturbance induced by the velocity oscillation. H_0 and H_{10} may be found by the molecular field theory or by the method of identification of system.

The interaction between the field and the magnetization yields a body force called Kelvin force [12] $f_m = \mu_0(M \cdot \nabla)H$, on the fluid. When M and H are both acting along the z-axis direction, the Kelvin force becomes $f_m = \mu_0 M(\partial H/\partial z)$.

The linear governing equations of the first-order acoustic quantities such as velocity and temperature of the fluid are given as following:

$$\rho_0 \frac{du'}{dt} = -\frac{\partial p'}{\partial z} + \mu_0 M_0 \frac{\partial H'}{\partial z} + \eta \nabla^2 u', \tag{3}$$

$$\rho_0 c\left(\frac{\partial T'}{\partial t} + u'\gamma\right) - \mu_0 T_0 k_1 \frac{dH'}{dt} = k\nabla^2 T' \tag{4}$$

with boundary conditions:

$$r = r_0, \quad u' = 0 \quad T' = 0, \tag{5}$$

where $T_0 = (T_h + T_c)/2$ is the mean temperature, $M_0 = \chi(H_e + H_{10})$ is the magnetization corresponding to the magnetic field $H_e + H_{10}$, $u, p, T, \eta, c, k, \rho_0, \mu_0$ are the velocity, the pressure, the temperature, the viscosity, the heat capacity, the thermal conductivity, the mean density and the permeability of free space, respectively. The sign "′" denotes the wave quantities. Here, the first kind boundary condition $T' = 0$ at $r = r_0$ is imposed on the tube wall [14]. The fluid properties, η, c, k, ρ_0, are taken as constants in this paper.

The flow is assumed to be entirely in the z-direction, and the pressure gradient can be found by using the approximation

$$\frac{\partial p'}{\partial z} = -p_0 e^{i(\omega t + \theta)}. \tag{6}$$

Eq. (6) is proposed by Watson [15], where θ is a phase shift between the pressure and the velocity, p_0 is related to not only the magnitude but also the phase angle of velocity oscillation (refer to following Eqs. (8) and (14)). Eq. (6) gives rise to a velocity distribution

$$u' = u(r)e^{i\omega t}. \tag{7}$$

Substituting Eqs. (2), (6) and (7) into the momentum equation (3) yields

$$v\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u(r)}{\partial r}\right)\right] - i\omega' u(r) = -\frac{p_0 e^{i\theta}}{\rho_0} \tag{8}$$

with

$$\omega' = \omega - \frac{k_1 \gamma \mu_0 M_0}{(1+\chi)\rho_0} i, \tag{9}$$

where $v = \eta/\rho_0$ is the kinematic viscosity and ω' is the complex angular frequency, respectively.

Considering a transformation on the dependent variable from $u(r)$ to $f_v(r)$, we express the distribution of velocity in the cross-section of the flow channel as

$$u(r) = u_0(1 - f_v), \tag{10}$$

where u_0 is the maximum centerline velocity and f_v is the viscous dissipation function, respectively. A more convenient form for $u(r)$ can be obtained by using the tidal displacement Δz instead of the maximum axial velocity u_0 in the Eq. (10). For the sinusoidal oscillations ($e^{i\omega t}$) considered here, we have [7]

$$\Delta z = (2u_0/\omega)|1 - \bar{f}_v| \tag{11}$$

with

$$\bar{f}_v = \frac{2\pi}{\pi r_0^2}\int_0^{r_0} f_v(r) r\, dr,$$

where the vertical bars representing the absolute value. Substituting Eq. (10) into Eq. (8) gives

$$u_0 v\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_v}{\partial r}\right) - \frac{i\omega'}{v} f_v\right] = -\frac{p_0 e^{i\theta}}{\rho_0} + i\omega' u_0. \tag{12}$$

This equation should be satisfied independently of transverse location r and kinematic viscosity v, we therefore have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_v}{\partial r}\right) - \frac{i\omega'}{v} f_v = 0 \tag{13}$$

and

$$-\frac{p_0 e^{i\theta}}{\rho_0} + i\omega' u_0 = 0. \tag{14}$$

Eq. (14) is a very close approximation to Eq. (8) at the entrance of the system (when the viscosity is ignored). It gives the relation between p_0 and u_0.

Solving Eq. (13) under the boundary condition $f_v(r_0) = 1$ yields

$$f_v = \frac{J_0\left[(i-1)\sqrt{\omega'\tau_v}(r/r_0)\right]}{J_0\left[(i-1)\sqrt{\omega'\tau_v}\right]}, \tag{15}$$

where $\tau_v = r_0^2/2v$ is the viscosity relaxation time and J_0 the Bessel function of first kind of order zero, respectively.

Substituting Eqs. (10) and (15) into Eq. (7) gives

$$u' = u_0\left(1 - \frac{J_0\left[(\mathrm{i}-1)\sqrt{\omega'\tau_v}(r/r_0)\right]}{J_0\left[(\mathrm{i}-1)\sqrt{\omega'\tau_v}\right]}\right)\mathrm{e}^{\mathrm{i}\omega t}. \tag{16}$$

The instantaneous temperature distribution T can assume the locally form

$$T = T_0 + \gamma z + T' = T_0 + \gamma[z + g(r)\mathrm{e}^{\mathrm{i}\omega t}] \tag{17}$$

with

$$T' = \gamma g(r)\,\mathrm{e}^{\mathrm{i}\omega t}. \tag{18}$$

Eq. (18) is proposed by Kurzweg and his coworkers [6,8].

Substituting Eqs. (2) and (18) into Eq. (4) yields the simplified form of the temperature equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) - \frac{\mathrm{i}\omega}{\alpha}g(r) = \left(\frac{1}{\alpha} + \frac{\mathrm{i}\omega\mu_0 T_0 H_0 k_1}{\gamma k}\right)u(r), \tag{19}$$

where $\alpha = k/(\rho_0 c)$ is the thermal diffusivity of the fluid.

In Eq. (19), we have neglected the second-order wave quantity and assumed $k_1\gamma l \ll (1+\chi)H_0$. Solution of Eq. (19) is

$$g(r) = E_1\left[\frac{\mathrm{i}}{\rho_0 c\omega}(1-f_\alpha) + \frac{Pr}{\mathrm{i}\rho_0 c(\omega P_r - \omega')}(f_v - f_\alpha)\right] \tag{20}$$

with

$$E_1 = u_0\left(\rho_0 c + \frac{\mathrm{i}\omega\mu_0 T_0 H_0 k_1}{\gamma}\right), \tag{21}$$

$$f_\alpha = \frac{J_0\left[(\mathrm{i}-1)\sqrt{\omega\tau_\alpha}(r/r_0)\right]}{J_0\left[(\mathrm{i}-1)\sqrt{\omega\tau_\alpha}\right]}, \tag{22}$$

where f_α is the thermal dissipation function, $\tau_\alpha = r_0^2/2\alpha$ is the thermal relaxation time and Pr is the Prandtl number, respectively.

To determine the axial flow of heat in the present system, we equate the total axial conduction plus convection heat flux to an effective thermal conductivity $\rho_0 c k_\mathrm{e}$ multiplied by the axial temperature gradient γ. Mathematically we have [8]

$$k_\mathrm{e}\gamma = \alpha\gamma - \frac{2\pi}{\pi r_0^2}\int_0^{r_0}\langle[u']_\mathrm{R}[T']_\mathrm{R}\rangle_t r\,\mathrm{d}r, \tag{23}$$

where the subscript R indicates the real part of the function in the bracket, k_e is the effective thermal diffusivity, and $\langle\rangle_t$ indicates the time average of the quantity in the bracket, respectively.

With the aid of Eqs. (7), (18), and (23), the effective diffusivity can be written as the following form:

$$k_\mathrm{e} = \alpha - \frac{1}{r_0^2}\int_0^{r_0}(g_\mathrm{r}u_\mathrm{r} + g_\mathrm{i}u_\mathrm{i})r\,\mathrm{d}r \tag{24}$$

where g_r and g_i indicate the real and imaginary part of the $g(r)$, u_r and u_i the real and imaginary part of the $u(r)$, respectively.

4. Results and discussion

The velocity function of transverse distribution $u(r)$: An example of the distribution of $u(r)$ is shown in Fig. 3. The spatial average over cross-sectional area of the $u(r)$ is shown in Fig. 4 with $t_v = \sqrt{\omega\tau_v}$, where $\omega\tau_v$ is a non-dimensional viscosity relaxation time. The $\langle u_\mathrm{r}\rangle_r$ and $\langle u_\mathrm{i}\rangle_r$ both increase along with the increasing of the t_v.

The temperature function of transverse distribution $g(r)$: An example of the distribution of $g(r)$ is shown in Fig. 5. The spatial average over cross-sectional area of the $g(r)$ is shown in Fig. 6 with $t_\alpha = \sqrt{\omega\tau_\alpha}$, where $\omega\tau_\alpha$ is a non-dimensional thermal relaxation time. The $\langle g_\mathrm{r}\rangle_r$ is negative with all value of the t_α and the $\langle g_\mathrm{i}\rangle_r$ is positive except for small t_α.

The effective thermal diffusivity k_e: Eq. (24) indicates that the effective thermal diffusivity k_e is proportional to Δz^2 and the k_e increases with increasing ω.

Let us define a non-dimensional effective thermal diffusivity $\lambda = k_\mathrm{e}/\alpha$. The calculated and experimental results of λ are shown in Fig. 7. The range of frequencies used was 1–5 Hz.

The external magnetic field induces a magnetization in the ferromagnetic fluid. In the magnetic equation of state the magnetization is a function of both magnetic field and temperature, so that the applied temperature

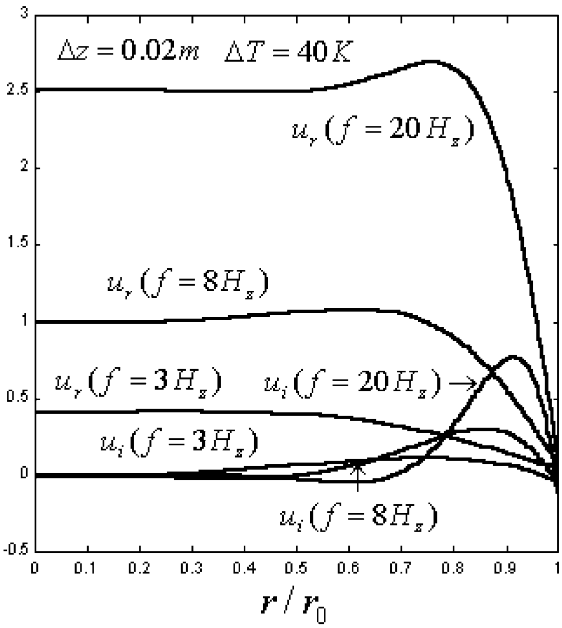

Fig. 3. Transverse distribution of $u(r)$.

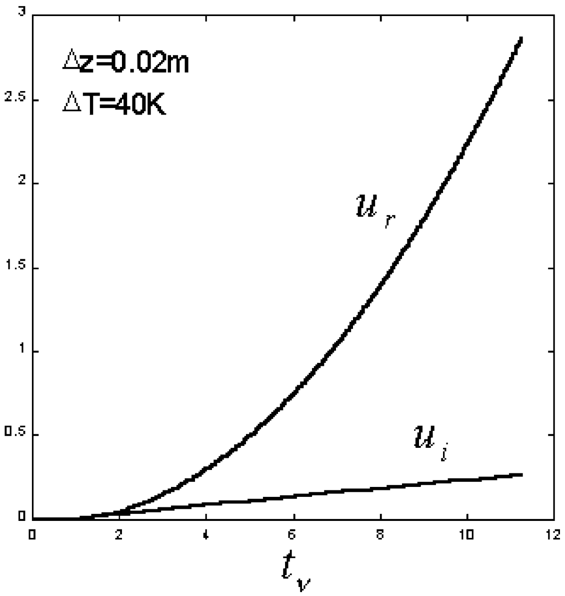

Fig. 4. The spatial average over cross-sectional area of the $u(r)$.

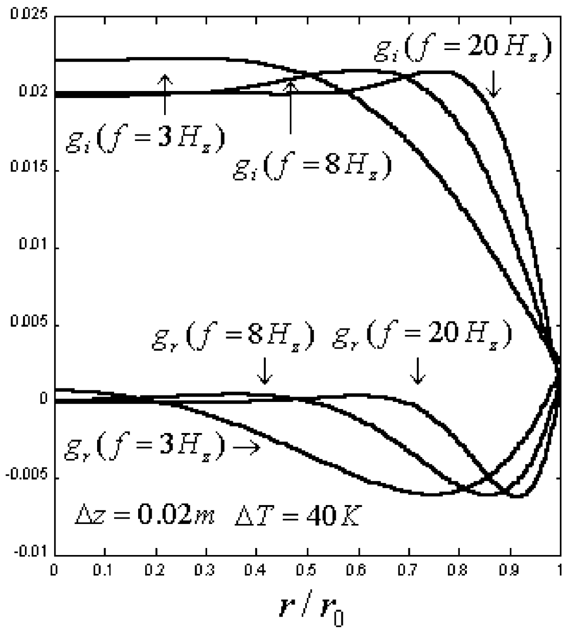

Fig. 5. Transverse distribution of $g(r)$.

gradient causes a spatial variation in the magnetization. The variation induces a non-uniform magnetic field, which is one of the driving forces causing the enhanced-heat transfer [12]. The variation of dimensionless effective thermal diffusivity λ with respect to magnetic induction B_e is shown in Fig. 8, where $B_e = \mu_0 H_e$ is the magnetic induction corresponding to the external magnetic field H_e.

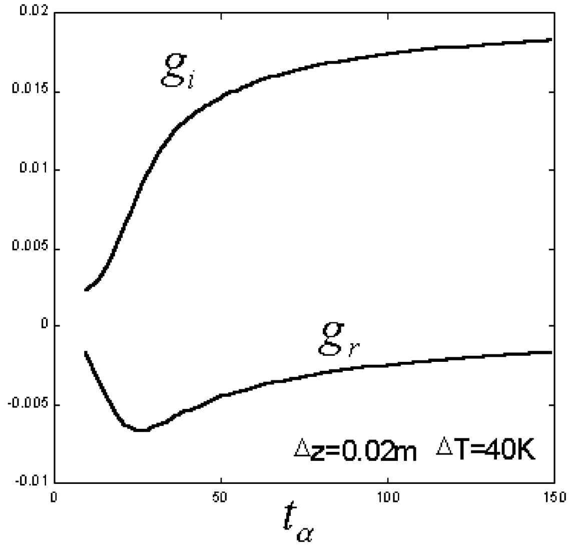

Fig. 6. The spatial average over cross-sectional area of the $g(r)$.

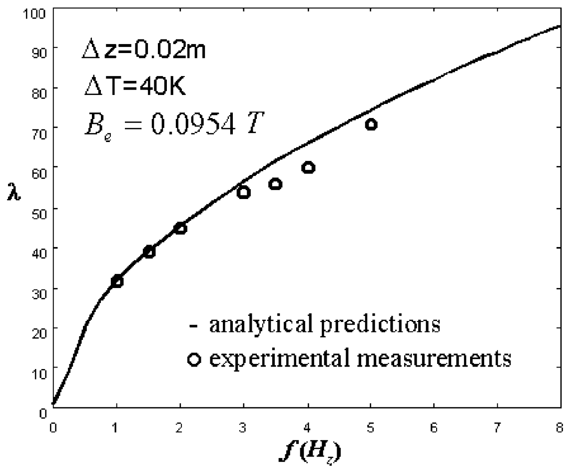

Fig. 7. The relation between a non-dimensional effective thermal diffusivity λ and the frequency f.

The conditions of the experiment and fluid properties are that $\Delta z = 0.02$ (m) and u_0 can be obtained by Eq. (11); $\Delta T = 40$ K; $B_e = 0.0954$ (T) and $M_0 = 3.0 \times 10^4$ (A/m) in Fig. 7; $f = 1$ Hz in Fig. 8; $k = 0.151$ (W/m K); $\rho_0 = 970$ kg/m^3 ; $c = 1745$ (J/kg K); $k_1 = 31$ (A/m K); $\eta = 0.015$ (kg/ms).

It is shown from Figs. 7 and 8 that λ is larger than one as long as H_e and ω are not equal to zero. It is therefore illustrated that both oscillation and magnetic are valuable to the heat transfer of ferromagnetic fluid, and that the influence of oscillation in conjunction with magnetic field on the enhanced-heat transfer for ferromagnetic fluid is greater than that of single oscillation or magnetic field.

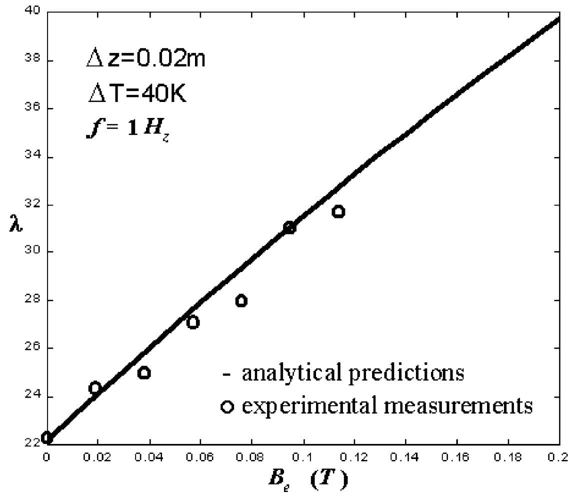

Fig. 8. The variation of dimensionless effective thermal diffusivity λ with respect to magnetic induction B_e.

Comparing Fig. 7 with Fig. 8, we can see that the affect of external magnetic field on λ is not so great as that of oscillating.

5. Conclusion

Enhanced-heat transfer of a ferromagnetic fluid in an external magnetic field has been analyzed in this paper. The theoretical expression for the effective thermal diffusivity is derived. A comparison of the analytical results with the experimental data shows a good agreement with a tolerance of less than 10%. The study of this paper shows that the dimensionless effective enhanced-thermal diffusivity λ of the longitudinal heat transport in the ferromagnetic fluid depends on oscillating frequency f and external magnetic field H_e. λ increases along with the increasing ω or the increasing H_e. Therefore, the heat transfer of a ferromagnetic fluid can be controlled by the presence of an acoustic field and a magnetic field.

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